Fuzzy Control

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Stick Balancing

Inverted pendulum
Typical Example: Cartpole Problem

Balance an upright standing pole

Lower end of pole can be moved unrestrained along horizontal axis.

Mass \( m \) at foot and mass \( M \) at head.

Influence of mass of shaft itself is negligible.

Determine force \( F \) (control variable) that is necessary to balance pole standing upright.

That is measurement of following output variables:

- angle \( \theta \) of pole in relation to vertical axis,
- change of angle, \( i.e. \) triangular velocity \( \dot{\theta} = \frac{d\theta}{dt} \)

Both should converge to zero.
Input variables $\xi_1, \ldots, \xi_n$, control variable $\eta$

Measurements: used to determine actual value of $\eta$

Assumption: $\xi_i$, $1 \leq i \leq n$ is value of $X_i$, $\eta \in Y$

Solution: control function $\varphi$

$$
\varphi : X_1 \times \ldots \times X_n \rightarrow Y \\
(x_1, \ldots, x_n) \mapsto y
$$
Angle $\theta \in X_1 = [-90^\circ, 90^\circ]$

Theoretically, every angle velocity $\dot{\theta}$ possible.

Extreme $\dot{\theta}$ are artificially achievable.

Assume $-45^\circ/s \leq \dot{\theta} \leq 45^\circ/s$ holds, i.e. $\dot{\theta} \in X_2 = [-45^\circ/s, 45^\circ/s]$.

Absolute value of force $|F| \leq 10\, \text{N}$.

Thus define $F \in Y = [-10\, \text{N}, 10\, \text{N}]$. 
Example: Cartpole Problem (cont.)

Differential equation of cartpole problem:

\[(M + m) \sin^2 \theta \cdot l \cdot \ddot{\theta} + m \cdot l \cdot \sin \theta \cos \theta \cdot \dot{\theta}^2 - (M + m) \cdot g \cdot \sin \theta = -F \cdot \cos \theta\]

Compute \(F(t)\) such that \(\theta(t)\) and \(\dot{\theta}(t)\) converge towards zero quickly.

Physical analysis demands knowledge about physical process.

In most real applications: No closed solution,
The standard is to use Runge Kutta Methods for systems of partial differential equations for approximate solutions
New successful methods are often nature inspired, such as Model-based Fuzzy Control,
Reinforcement Learning using data, evolutionary optimisation techniques
Problems of Classical Approach

Often very difficult or even impossible to specify accurate mathematical model.

Description with differential equations is very complex.

Profound physical knowledge from engineer.

Exact solution can be very difficult.

Should be possible: to control process without physical-mathematical model. Human being knows how to balance a stick or to ride bike without knowing existence of differential equations.
Simulate behavior of human who knows how to control.

That is a **knowledge-based analysis**.

Directly ask expert to perform analysis.

Then expert specifies knowledge as **linguistic rules**, *e.g.* for cartpole problem:

“If $\theta$ is approximately zero and $\dot{\theta}$ is also approximately zero, then $F$ has to be approximately zero, too.”

The aim is to find a simple solution that is **good enough**.

With further steps (using data and learning methods) the solutions is refined, if necessary.
1. Formulate set of linguistic rules:

Determine linguistic terms (represented by fuzzy sets). $X_1, \ldots, X_n$ and $Y$ is partitioned into fuzzy sets. Define $p_1$ distinct fuzzy sets $\mu_1^{(1)}, \ldots, \mu_{p_1}^{(1)} \in \mathcal{F}(X_1)$ on set $X_1$. Associate linguistic term with each set.
Coarse and Fine Fuzzy Partitions

- negative
- approx. zero
- positive

- neg. big
- neg. small
- pos. small
- pos. big

- neg. medium
- approx. zero
- pos. medium

-90 0 90
$X_1$ corresponds to interval $[a, b]$ of real line, \( \mu_1^{(1)}, \ldots, \mu_p^{(1)} \in \mathcal{F}(X_1) \) are triangular functions

\[
\mu_{x_0, \varepsilon} : [a, b] \rightarrow [0, 1]
\]

\[
x \mapsto 1 - \min\{ \varepsilon \cdot |x - x_0|, 1 \}.
\]

If $a < x_1 < \ldots < x_p < b$, only $\mu_2^{(1)}, \ldots, \mu_p^{(1)}$ are triangular. Boundaries are treated differently.
left fuzzy set:

\[ \mu^{(1)}_{1} : [a, b] \rightarrow [0, 1] \]

\[ x \mapsto \begin{cases} 
1, & \text{if } x \leq x_1 \\
1 - \min\{\varepsilon \cdot (x - x_1), 1\} & \text{otherwise}
\end{cases} \]

right fuzzy set:

\[ \mu^{(1)}_{\rho_1} : [a, b] \rightarrow [0, 1] \]

\[ x \mapsto \begin{cases} 
1, & \text{if } x_{\rho_1} \leq x \\
1 - \min\{\varepsilon \cdot (x_{\rho_1} - x), 1\} & \text{otherwise}
\end{cases} \]
Example: Cartpole Problem (cont.)

$X_1$ partitioned into 7 fuzzy sets.

Similar fuzzy partitions for $X_2$ and $Y$.

**Next step:** specify rules

if $\xi_1$ is $A^{(1)}$ and \ldots and $\xi_n$ is $A^{(n)}$ then $\eta$ is $B$,

$A^{(1)}, \ldots, A^{(n)}$ and $B$ represent linguistic terms corresponding to $\mu^{(1)}, \ldots, \mu^{(n)}$ and $\mu$ according to $X_1, \ldots, X_n$ and $Y$.

Rule base consists of $k$ rules.
Example: Cartpole Problem (cont.)

19 rules for cartpole problem, it is not necessary to determine all table entries. A table entry is interpreted as a rule:
If $\theta$ is approximately zero and $\dot{\theta}$ is negative medium then $F$ is positive medium.
Mamdani Controller
Architecture of a Mamdani Fuzzy Controller

- Fuzzification Interface
- Decision Logic
- Defuzzification Interface
- Knowledge Base
- Measured Values
- Controller Output
- Not Fuzzy
- Not Fuzzy

Not fuzzy values are not fuzzy.
19 rules for cartpole problem:
If $\theta$ is *approximately zero* and $\dot{\theta}$ is *negative medium* then $F$ is *positive medium*.
Evaluation of a single rule

if \( \text{positive small} \) and \( \approx \text{approx. zero} \), then \( \text{positive small} \).
Evaluation of a single rule

Rule evaluation for Mamdani-Assilian controller.

Input tuple (25, -4) leads to fuzzy output.

Crisp output is determined by defuzzification.

\( \theta = 25 \)  
\( \dot{\theta} = -4 \)  

Input

\( F = ? \)  
Output
Evaluation of several rules

Rule evaluation for Mamdani-Assilian controller.

Input tuple \((25, -4)\) leads to fuzzy output.

Crisp output is determined by defuzzification.
Definition of Table-based Control Function I

Given is the measurement \((x_1, \ldots, x_n) \in X_1 \times \ldots \times X_n\)

Consider a rule \(R\)

\[
\text{if } \mu^{(1)} \text{ and } \ldots \text{ and } \mu^{(n)} \text{ then } \eta.
\]

The fuzzyfication unit computes for the input \((x_1, \ldots, x_n)\) a „degree of fulfillment“ of the premise of the rule:

For \(1 \leq v \leq n\), the membership degree \(\mu^{(v)}(x_v)\) is calculated. The \(n\) degrees are combined conjunctively with the min-operator and give the fulfillment degree \(\alpha\)

For each rule \(R_r\) with \(1 \leq r \leq k\), compute the fulfillment degree \(\alpha_r\)
For the input \((x_1, \ldots, x_n)\) and a rule \(R\) the decision unit calculates the output

\[
\mu_{x_1, \ldots, x_n}^{\text{output}(R)} : Y \rightarrow [0, 1], \\
y \mapsto \min \left( \mu^{(1)}(x_1), \ldots, \mu^{(n)}(x_n), \eta(y) \right).
\]
The decision logic combines the output fuzzy sets from all rules $R_1,...,R_k$ by using the or-operator \textbf{maximum}. This results in the output fuzzy set

$$\mu^{\text{output}}_{x_1,...,x_n} : Y \rightarrow [0, 1]$$

Then $\mu^{\text{output}}_{x_1,...,x_n}$ is passed to defuzzification interface.
Defuzzification interface derives crisp value from $\mu^{\text{output}}_{x_1,\ldots,x_n}$.

Most common **defuzzification** methods:

- max criterion,
- mean of maxima,
- center of gravity.

See Google Patents at defuzzification: More than 1080 methods
Center of Gravity (COG) Method

Same preconditions as MOM method.

\( \eta = \text{center of gravity/area of } \mu_{x_1, \ldots, x_n}^{\text{output}} \)

If \( Y \) is finite, then

\[
\eta = \frac{\sum_{y_i \in Y} y_i \cdot \mu_{x_1, \ldots, x_n}^{\text{output}}(y_i)}{\sum_{y_i \in Y} \mu_{x_1, \ldots, x_n}^{\text{output}}(y_i)}.
\]

If \( Y \) is infinite, then

\[
\eta = \frac{\int_{y \in Y} y \cdot \mu_{x_1, \ldots, x_n}^{\text{output}}(y) \, dy}{\int_{y \in Y} \mu_{x_1, \ldots, x_n}^{\text{output}}(y) \, dy}.
\]
Task: compute $\eta_{\text{COG}}$ and $\eta_{\text{MOM}}$ of fuzzy set shown below.
Example for COG
Continuous and Discrete Output Space

\[ \eta_{\text{COG}} = \frac{\int_{0}^{10} y \cdot \mu_{x_1, \ldots, x_n}^{\text{output}}(y) \, dy}{\int_{0}^{10} \mu_{x_1, \ldots, x_n}^{\text{output}}(y) \, dy} \]

\[ = \frac{\int_{0}^{5} 0.4y \, dy + \int_{5}^{7} (0.2y - 0.6)y \, dy + \int_{7}^{10} 0.8y \, dy}{5 \cdot 0.4 + 2 \cdot \frac{0.8 + 0.4}{2} + 3 \cdot 0.8} \]

\[ \approx \frac{38.7333}{5.6} \approx 6.917 \]
Problem Cases for MOM and COG

Stone on the street, the car should not use COG (giving 0) but -1 or +1
Mamdani Control Applications
Example: Engine Idle Speed Control
VW 2000cc 116hp Motor (Golf GTI)
20.8.92 MW(4500MP):827.13UpM
Structure of the Fuzzy Controller

fuzzy controller

- meta controller
  - AARSREV
  - REV0_LO
  - dAIRCON
- data prep.
- state detect. and MFC activ.
- dREV
- dAARCUR
- control range limit.
- pilot value for air conditioning system

AARCURIN
Deviation of the Number of Revolutions   \( d_{REV} \)
Gradient of the Number of Revolutions  gREV
Change of Current for Auxiliary Air Regulator  dAARCUR
If the deviation from the desired number of revolutions is negative small \textbf{and} the gradient is negative medium, \textbf{then} the change of the current for the auxiliary air regulation should be positive medium.
If the deviation from the desired number of revolutions is negative small and the gradient is negative medium, then the change of the current for the auxiliary air regulation should be positive medium.

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Performance Characteristics
M151B1  Fz  SR  Kupplung
18.8.92

Drehzahl [UpM]

800  1000  1200  1400  1600  1800

Meßpunkt

800  1000  1200  1400  1600  1800

LFRSTRAUS

DRZ0...LO  LFRSTRAUS  LFRSDRE
Example: Automatic Gear Box AG4

Idea: car “watches” driver and classifies him/her into calm, normal, sportive (assign sport factor [0, 1]), or nervous (calm down driver).

Test car: different drivers, classification by expert (passenger).
Simultaneous measurement of 14 attributes, e.g., speed, position of accelerator pedal, speed of accelerator pedal, kick down, steering wheel angle.
Example: Automatic Gear Box

Continuously Adapting Gear Shift Schedule in VW New Beetle

![Diagram showing the classification of driver/driving situation by fuzzy logic and gear shift computation process.]

- **Fuzzification**
  - Accelerator pedal
  - Filtered speed of accelerator pedal
  - Number of changes in pedal direction
  - Sport factor [t-1]

- **Inference Machine**
  - Rule base

- **Defuzzification**
  - Sport factor [t]

- **Interpolation**
  - Determination of speed limits for shifting into higher or lower gear depending on sport factor

- **Gear Selection**
Example: Automatic Gear Box

Technical Details

Optimized program on Digimat:
24 byte RAM
702 byte ROM
uses 7 Mamdani fuzzy rules

Runtime: 80ms
12 times per second new sport factor is assigned.

In Series Line
Takagi Sugeno Control
Proposed by Tomohiro Takagi and Michio Sugeno.
Modification/extension of Mamdani controller.

Both in common: fuzzy partitions of input domain $X_1, \ldots, X_n$.

Difference to Mamdani controller:
- no fuzzy partition of output domain $Y$, no defuzzification
- controller rules $R_1, \ldots, R_k$ are given by

$$ R_r : \text{if } \xi_1 \text{ is } A_{i_1,r}^{(1)} \text{ and } \ldots \text{ and } \xi_n \text{ is } A_{i_n,r}^{(n)} \text{ then } \eta_r = f_r(\xi_1, \ldots, \xi_n), $$

$$ f_r : X_1 \times \ldots \times X_n \rightarrow Y. $$

- Typically, $f_r$ is linear, i.e. $f_r(x_1, \ldots, x_n) = a_0^{(r)} + \sum_{i=1}^{n} a_i^{(r)} x_i$. 
For given input \((x_1, \ldots, x_n)\) and for each \(R_r\), decision logic computes truth value \(\alpha_r\) of each premise, and then \(f_r(x_1, \ldots, x_n)\).

Analogously to Mamdani controller:

\[
\alpha_r = \min \left\{ \mu_{i_1,r}^{(1)}(x_1), \ldots, \mu_{i_n,r}^{(n)}(x_n) \right\}.
\]

Output equals crisp control value

\[
\eta = \frac{\sum_{r=1}^{k} \alpha_r \cdot f_r(x_1, \ldots, x_n)}{\sum_{r=1}^{k} \alpha_r}.
\]

Thus no defuzzification method necessary.
Example

\[ R_1 : \text{if } \xi_1 \text{ is } \frac{3}{9} \text{ then } \eta_1 = 1 \cdot \xi_1 + 0.5 \cdot \xi_2 + 1 \]

\[ R_2 : \text{if } \xi_1 \text{ is } \frac{3}{9} \text{ and } \xi_2 \text{ is } \frac{4}{13} \text{ then } \eta_2 = -0.1 \cdot \xi_1 + 4 \cdot \xi_2 + 1.2 \]

\[ R_3 : \text{if } \xi_1 \text{ is } \frac{3}{9} \frac{11}{18} \text{ and } \xi_2 \text{ is } \frac{4}{13} \text{ then } \eta_3 = 0.9 \cdot \xi_1 + 0.7 \cdot \xi_2 + 9 \]

\[ R_4 : \text{if } \xi_1 \text{ is } \frac{11}{18} \text{ and } \xi_2 \text{ is } \frac{4}{13} \text{ then } \eta_4 = 0.2 \cdot \xi_1 + 0.1 \cdot \xi_2 + 0.2 \]

If a certain clause "\( x_j \text{ is } A_{ij,r}^{(j)} \)" in rule \( R_r \) is missing, then \( \mu_{i_{j,r}}(x_j) \equiv 1 \) for all linguistic values \( i_{j,r} \).

For instance, here \( x_2 \) in \( R_1 \), so \( \mu_{i_{2,1}}(x_2) \equiv 1 \) for all \( i_{2,1} \).
Example

\[ R_1 : \text{if } \xi_1 \text{ is } 3 \text{ and } 9 \text{ then } \eta_1 = 1 \cdot \xi_1 + 0.5 \cdot \xi_2 + 1 \]

\[ R_2 : \text{if } \xi_1 \text{ is } 3 \text{ and } 9 \text{ and } \xi_2 \text{ is } 4 \text{ and } 13 \text{ then } \eta_2 = -0.1 \cdot \xi_1 + 4 \cdot \xi_2 + 1.2 \]

\[ R_3 : \text{if } \xi_1 \text{ is } 3 \text{ and } 9 \text{ and } 11 \text{ and } 18 \text{ and } \xi_2 \text{ is } 4 \text{ and } 13 \text{ then } \eta_3 = 0.9 \cdot \xi_1 + 0.7 \cdot \xi_2 + 9 \]

\[ R_4 : \text{if } \xi_1 \text{ is } 11 \text{ and } 18 \text{ and } \xi_2 \text{ is } 4 \text{ and } 13 \text{ then } \eta_4 = 0.2 \cdot \xi_1 + 0.1 \cdot \xi_2 + 0.2 \]

**input:** \((6, 7)\)

If a certain clause “\(x_j \text{ is } A_{ij,r}(j)\)” in rule \(R_r\) is missing, then \(\mu_{ij,r}(x_j) \equiv 1\) for all linguistic values \(i_{j,r}\).

For instance, here \(x_2\) in \(R_1\), so \(\mu_{i_{2,1}}(x_2) \equiv 1\) for all \(i_{2,1}\).
Example: Output Computation

input: \((\xi_1, \xi_2) = (6, 7)\)

\[
\begin{align*}
\alpha_1 &= \frac{1}{2} \land 1 = \frac{1}{2} & \eta_1 &= 6 + \frac{7}{2} + 1 = 10.5 \\
\alpha_2 &= \frac{1}{2} \land \frac{2}{3} = 1/2 & \eta_2 &= -0.6 + 28 + 1.2 = 28.6 \\
\alpha_3 &= \frac{1}{2} \land \frac{1}{3} = \frac{1}{3} & \eta_3 &= 0.9 \cdot 6 + 0.7 \cdot 7 + 9 = 19.3 \\
\alpha_4 &= 0 \land \frac{1}{3} = 0 & \eta_4 &= 6 + \frac{7}{2} + 1 = 10.5
\end{align*}
\]

output: \(\eta = f(6, 7) = \frac{1/2 \cdot 10.5 + 1/2 \cdot 28.6 + 1/3 \cdot 19.3}{1/2 + 1/2 + 1/3} = 19.5\)
Pass a bend with a car at constant speed.

Measured inputs:
- $\xi_1$ : distance of car to beginning of bend
- $\xi_2$ : distance of car to inner barrier
- $\xi_3$ : direction (angle) of car
- $\xi_4$ : distance of car to outer barrier

$\eta$ = rotation speed of steering wheel

$X_1 = [0\text{cm}, 150\text{cm}]$, $X_2 = [0\text{cm}, 150\text{cm}]$

$X_3 = [-90^\circ, 90^\circ]$, $X_4 = [0\text{cm}, 150\text{cm}]$
Fuzzy Partitions of $X_1$ and $X_2$
Fuzzy Partitions of $X_3$ and $X_4$

![Graph showing fuzzy partitions with labels outwards, 1, forward, inwards, small, and 40 on the axes.](image-url)
Rules for Car

\[ R_r : \text{if } \xi_1 \text{ is } A \text{ and } \xi_2 \text{ is } B \text{ and } \xi_3 \text{ is } C \text{ and } \xi_4 \text{ is } D \]

\[ \text{then } \eta = p_0^{(A,B,C,D)} + p_1^{(A,B,C,D)} \cdot \xi_1 + p_2^{(A,B,C,D)} \cdot \xi_2 \]
\[ + p_3^{(A,B,C,D)} \cdot \xi_3 + p_4^{(A,B,C,D)} \cdot \xi_4 \]

\[ A \in \{\text{small, medium, big}\} \]
\[ B \in \{\text{small, big}\} \]
\[ C \in \{\text{outwards, forward, inwards}\} \]
\[ D \in \{\text{small}\} \]
\[ p_0^{(A,B,C,D)}, \ldots, p_4^{(A,B,C,D)} \in \mathbb{R} \]
## Control Rules for the Car

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<th>$\xi_3$</th>
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<td>-0.100</td>
<td>0.000</td>
</tr>
<tr>
<td>$R_{15}$</td>
<td>big</td>
<td>small</td>
<td>outwards</td>
<td>-</td>
<td>0.370</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>$R_{16}$</td>
<td>big</td>
<td>small</td>
<td>forward</td>
<td>-</td>
<td>-0.900</td>
<td>0.000</td>
<td>0.034</td>
<td>-0.030</td>
<td>0.000</td>
</tr>
<tr>
<td>$R_{17}$</td>
<td>big</td>
<td>small</td>
<td>inwards</td>
<td>-</td>
<td>-1.500</td>
<td>0.000</td>
<td>0.005</td>
<td>-0.100</td>
<td>0.000</td>
</tr>
<tr>
<td>$R_{18}$</td>
<td>big</td>
<td>big</td>
<td>outwards</td>
<td>-</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.013</td>
<td>0.000</td>
</tr>
<tr>
<td>$R_{19}$</td>
<td>big</td>
<td>big</td>
<td>forward</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.006</td>
<td>0.000</td>
</tr>
<tr>
<td>$R_{20}$</td>
<td>big</td>
<td>big</td>
<td>inwards</td>
<td>-</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.010</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Assume that the car is $10 \text{ cm}$ away from beginning of bend ($\xi_1 = 10$). The distance of the car to the inner barrier be $30 \text{ cm}$ ($\xi_2 = 30$). The distance of the car to the outer barrier be $50 \text{ cm}$ ($\xi_4 = 50$). The direction of the car be “forward” ($\xi_3 = 0$).

Then according to all rules $R_1, \ldots, R_{20}$, only premises of $R_4$ and $R_7$ have a value $\neq 0$. 

# Membership Degrees to Control Car

<table>
<thead>
<tr>
<th>( \xi_1 = 10 )</th>
<th>small</th>
<th>medium</th>
<th>big</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.8</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \xi_2 = 30 )</th>
<th>small</th>
<th>big</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25</td>
<td>0.167</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>( \xi_3 = 0 )</th>
<th>outwards</th>
<th>forward</th>
<th>inwards</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \xi_4 = 50 )</th>
<th>small</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
For the premise of $R_4$ and $R_7$, $\alpha_4 = \frac{1}{4}$ and $\alpha_7 = \frac{1}{6}$, resp.

The rules weights $\alpha_4 = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{6}} = \frac{3}{5}$ for $R_4$ and $\alpha_5 = \frac{2}{5}$ for $R_7$.

$R_4$ yields

$$\eta_4 = 0.303 - 0.026 \cdot 10 + 0.061 \cdot 30 - 0.050 \cdot 0 + 0.000 \cdot 50$$
$$= 1.873.$$

$R_7$ yields

$$\eta_7 = 2.990 - 0.017 \cdot 10 + 0.000 \cdot 30 - 0.021 \cdot 0 + 0.000 \cdot 50$$
$$= 2.820.$$

The final value for control variable is thus

$$\eta = \frac{3}{5} \cdot 1.873 + \frac{2}{5} \cdot 2.820 = 2.2518.$$
Fuzzy Control

Biggest success of fuzzy systems in industry and commerce.

Special kind of model-based non-linear control method.

Examples: technical systems

- Electrical engine moving an elevator,
- Heating installation

Goal: define certain behavior

- Engine should maintain certain number of revolutions per minute.
- Heating should guarantee certain room temperature.