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1. Exercise Sheet

Exercise 1 Expected Payoffs

Consider a game with the following payoff matrix:

$$A = \begin{matrix} \\ C \\ D \end{matrix} \begin{matrix} \begin{matrix} C & D \end{matrix} \\ \begin{bmatrix} 0,0 & 1,1 \\ 0,0 & 1,1 \end{bmatrix} \end{matrix}$$

a) Compute the expected payoff for player 1 playing cooperate (C), when it is already known that player 2 plays a mixed strategy of $x = (1/2, 1/2)$.

b) Compute the expected payoff of player 1, when he plays a mixed strategy of $x_1 = (1/2, 1/2)$ and player 2 plays a mixed strategy of $x_2 = (1/2, 1/2)$.

c) Assume that both players currently play a mixed strategy of $x = (1/2, 1/2)$. What kind of strategy can player 1 play to increase his payoff? What kind of strategy can player 2 play to increase his payoff?

Exercise 2 Nash Equilibria

Consider a 2-Player Prisoner's Dilemma game with the following payoff matrix.

$$A = \begin{matrix} \\ C \\ D \end{matrix} \begin{matrix} \begin{matrix} C & D \end{matrix} \\ \begin{bmatrix} 3,3 & 0,5 \\ 5,0 & 1,1 \end{bmatrix} \end{matrix}$$

What strategy profile is a Nash-Equilibrium?

Exercise 3 Nash Equilibria

Roger and Colleen play a game. Each one has a coin. They will both show a side of their coin simultaneously. If both show heads, no money will be exchanged. If Roger shows heads and Colleen shows tails then Colleen will give Roger 1 Dollar. If Roger shows tails and Colleen shows heads, then Roger will pay Colleen 1 Dollar. If both show tails, then they both get 2 Dollar.

a) Write the payoff matrix (for both players). Note: You can write in one matrix or in two matrices.

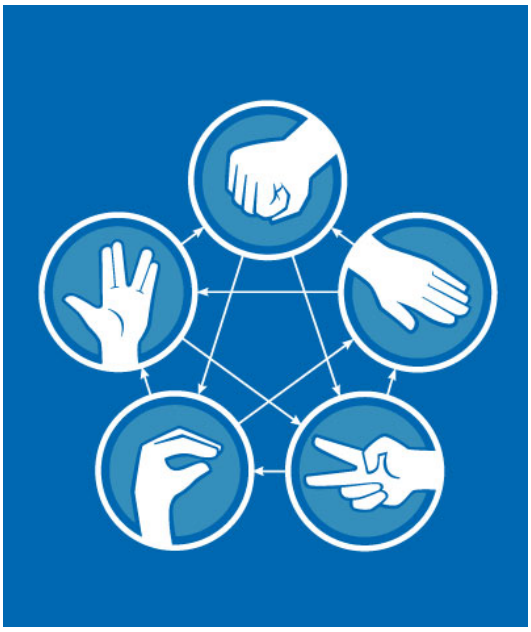
b) What is the best response of Colleen to Roger, when he plays/shows tails?

c) What is the Nash-equilibrium for this payoff matrix?

Exercise 4 Nash Equilibria in Mixed Strategies

Consider the game Rock-Paper-Scissors-Lizard-Spock. The rules are defined as:

- Scissors cuts Paper
- Paper covers Rock
- Rock crushes Lizard
- Lizard poisons Spock
- Spock smashes Scissors
- Scissors decapitates Lizard
- Lizard eats Paper
- Paper disproves Spock
- Spock vaporizes Rock
- Rock crushes Scissors



a) Write the payoff matrix for the 2-player version of the game.

b) Show that there cannot be a Nash-Equilibrium with pure strategies.

c) Proof that the Nash-Equilibrium is a mixed strategy with the probability distribution $x = (1/5, 1/5, 1/5, 1/5, 1/5)$.

- Hint: read up on the proof for the base-game Rock-Paper-Scissors and adapt it to the 5 strategy variant: `https://oyc.yale.edu/sites/default/files/mixed_strategies_handout_0_0.pdf`

Exercise 5 Replicator Equations and Fixed Points

Consider the Stag-Hunt game with the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} \text{C} & \text{D} \end{array} \\ \begin{array}{c} \text{C} \\ \text{D} \end{array} & \begin{bmatrix} 2, 2 & 0, 1 \\ 1, 0 & 1, 1 \end{bmatrix} \end{array}$$

a) Use replicator equations to calculate the fixed point(s), for the number of cooperators.

b) Show to which fixed point the population converges if the initial frequency of cooperators is 75%.

c) Show which fixed point the population converges to for any start-frequency of cooperators.