Bayes Networks Winter 2018/2019

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Exercise Sheet 2

Exercise 5 Conditional Probabilities

a) Four balls are placed into four boxes one after another. All 4^4 orders be equally likely. What is the probability that a box contains exactly three balls given the fact that the first two balls have been placed into different boxes?

b) A family has two children. What is the probability of both being girls if is known that at least one of them is a girl?

c) What is the probability of b) if it is known that the younger child is a girl?

Exercise 6 Stochastic Independency

a) A wheel of fortune has 36 numbered sectors (numbers 1 to 36). These sectors are colored in red (R) or blue (B) according the following scheme:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
R	R	R	R	R	B	B	B	B	R	R	R	R	B	B	B	B	B
36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19

We consider the three events

A : the wheel stops in a red sector,
B : the wheel stops in a sector with an even number,
C : the wheel stops in a sector with a number ≤ 18.

Show that these events are pairwise independent but that $P(A \cap B \cap C) = P(A)P(B)P(C)$ does not hold.

b) Two fair dice, red and white, are cast. We consider the following three events:

A : the red die shows up 1 or 2,
B : the white die shows up 3, 4 or 5,
C : the sum of the spots of both dice is 4, 11 or 12.

Show that $P(A \cap B \cap C) = P(A)P(B)P(C)$ holds but not the pairwise independence.

Exercise 7 Marginal Distributions, (conditional) Independencies

Consider the contingency table with attributes M=Malaria, G=Flu, F=Fever and H=cough. The respective binary domains $\text{dom}(\mathsf{X}) = \{\mathsf{x}, \overline{\mathsf{x}}\}$ designate for $\mathsf{X} = \mathsf{M}, \mathsf{G}, \mathsf{F}, \mathsf{H}$ the meaning $\mathsf{x} \mathrel{\widehat{=}}$ "symptom/desease is present" and $\overline{\mathsf{x}} \mathrel{\widehat{=}}$ "symptom/desease is not present".

p_{MGFH}		$\mathsf{G}=\mathsf{g}$, $\mathsf{M}=\mathsf{m}$	$\mathsf{G}=\mathsf{g}$, $\mathsf{M}=\overline{\mathsf{m}}$	$\mathsf{G}=\overline{\mathsf{g}}$, $\mathsf{M}=\mathsf{m}$	$\mathsf{G}=\overline{\mathsf{g}}$, $\mathsf{M}=\overline{\mathsf{m}}$
$\mathsf{F}=\mathsf{f}$	$\mathsf{H}=\mathsf{h}$	144	1008	192	216
	$\mathsf{H}=\overline{\mathsf{h}}$	36	252	448	504
$\mathsf{F}=\overline{\mathsf{f}}$	$\mathsf{H}=\mathsf{h}$	16	432	48	1944
	$\mathsf{H}=\overline{\mathsf{h}}$	4	108	112	4536

a) Compute all four marginal distributions.

b) Compute $P(\mathsf{M}=\mathsf{m} \mid \mathsf{F}=\mathsf{f})$ and $P(\mathsf{G}=\mathsf{g} \mid \mathsf{F}=\mathsf{f})$, as well as $P(\mathsf{M}=\mathsf{m} \mid \mathsf{F}=\mathsf{f}, \mathsf{H}=\mathsf{h})$ and $P(\mathsf{G}=\mathsf{g} \mid \mathsf{F}=\mathsf{f}, \mathsf{H}=\mathsf{h})$.

c) Show that M and G are marginally independent, but conditionally dependent given F.

d) Show that F and H are marginally dependent, but conditionally independent given G.

Exercise 8 Simpson Paradoxon

The equal opportunities officer of the university fantasia was ordered to analyse the skewed distribution of scholarships per gender. All faculty scholarship boards report an equal share of scholarships per gender, however the scholarships of the whole university sum up to:

	Number of applications	Number of scholarships
female	3000	378
male	3000	1299
total	6000	1677

How can the differing observations of the equal opportunities officer and the scholarship boards be explained? Present scholarships distributions of at least two faculties which yield such an result.

Additional Exercise Probabilities: Triple Duel

A, B and C compete against each other in a duel with pistols simultaneously. A is the worst shooter: His chance of hitting the target is 0.3. The chance of C is 0.5 whereas B never misses his target. The three shot in order A, B, C, A, ... at a target of their choice (of course, who was shot, quits the game) until just one of them is left. Which is A's best strategy?